Paper Reference(s)

6669/01

Edexcel GCE

Further Pure Mathematics FP3

Advanced/Advanced Subsidiary

Monday 24 June 2013 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P43143A

1. A hyperbola H has equation

$$\frac{x^2}{a^2} - \frac{y^2}{25} = 1, \qquad \text{where } a \text{ is a positive constant.}$$

The foci of H are at the points with coordinates (13, 0) and (–13, 0).

Find

(*a*) the value of the constant a,

(3)

(*b*) the equations of the directrices of H.

(3)

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2. (*a*) Find

$$\int \frac{1}{\sqrt{(4x^2+9)}}\, dx$$

(2)

(*b*) Use your answer to part (*a*) to find the exact value of

$$\int_{-3}^{3} \frac{1}{\sqrt{(4x^2+9)}}\, dx$$

giving your answer in the form $k \ln(a + b\sqrt{5})$, where a and b are integers and k is a constant.

(3)

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3. The curve with parametric equations

$$x = \cosh 2\theta, \quad y = 4 \sinh \theta, \quad 0 \leq \theta \leq 1$$

is rotated through 2π radians about the x-axis.

Show that the area of the surface generated is $\lambda(\cosh^3 \alpha - 1)$, where $\alpha = 1$ and λ is a constant to be found.

(7)

4.

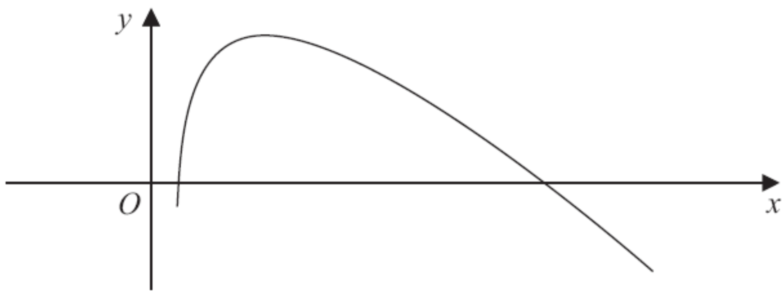

Figure 1

Figure 1 shows part of the curve with equation

$$y = 40 \operatorname{arcosh} x - 9x, \qquad x \geq 1$$

Use calculus to find the exact coordinates of the turning point of the curve, giving your answer in the form $\left(\frac{p}{q}, r\ln 3 + s\right)$ where p, q, r and s are integers.

(7)

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5. The matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & a \\ 2 & b & c \\ -1 & 0 & 1 \end{pmatrix}, \text{ where } a, b \text{ and } c \text{ are constants.}$$

(*a*) Given that **j** + **k** and **i** – **k** are two of the eigenvectors of **M**,

find

(i) the values of a, b and c,

(ii) the eigenvalues which correspond to the two given eigenvectors.

(8)

(*b*) The matrix **P** is given by

$$\mathbf{P} = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & d \\ -1 & 0 & 1 \end{pmatrix}, \text{ where } d \text{ is constant, } d \neq -1$$

Find

(i) the determinant of **P** in terms of d,

(ii) the matrix $\mathbf{P}^{-1}$ in terms of d.

(5)

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6. Given that

$$I_n = \int_0^4 x^n \sqrt{(16 - x^2)}\,dx, \qquad n \geq 0,$$

(*a*) prove that, for $n \geq 2$,

$$(n + 2)I_n = 16(n - 1)I_{n-2}$$

(6)

(*b*) Hence, showing each step of your working, find the exact value of I_5.

(5)

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7. The ellipse E has equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \qquad a > b > 0$$

The line l is a normal to E at a point $P(a\cos\theta, b\sin\theta)$, $\quad 0 < \theta < \frac{\pi}{2}$

(*a*) Using calculus, show that an equation for l is

$$ax\sin\theta - by\cos\theta = (a^2 - b^2)\sin\theta\cos\theta$$

(5)

The line l meets the x-axis at A and the y-axis at B.

(*b*) Show that the area of the triangle OAB, where O is the origin, may be written as $k\sin 2\theta$, giving the value of the constant k in terms of a and b.

(4)

(*c*) Find, in terms of a and b, the exact coordinates of the point P, for which the area of the triangle OAB is a maximum.

(3)

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8. The plane Π_1 has vector equation

$$\mathbf{r}.(3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) = 5$$

(*a*) Find the perpendicular distance from the point (6, 2, 12) to the plane Π_1.

(3)

The plane Π_2 has vector equation

$$\mathbf{r} = \lambda(2\mathbf{i} + \mathbf{j} + 5\mathbf{k}) + \mu(\mathbf{i} - \mathbf{j} - 2\mathbf{k}), \text{ where } \lambda \text{ and } \mu \text{ are scalar parameters.}$$

(*b*) Find the acute angle between Π_1 and Π_2 giving your answer to the nearest degree.

(5)

(*c*) Find an equation of the line of intersection of the two planes in the form $\mathbf{r} \times \mathbf{a} = \mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors.

(6)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme		Marks
	Mark (a) and (b) together		
1. (a) & (b)	$ae = 13$ **and** $a^2(e^2 - 1) = 25$	Sight of **both** of these (can be implied by their work) (allow ± ae = ±13 or ± ae = 13 or ae = ±13)	B1
	Solves to obtain $a^2 = ...$ or $a = ...$	Eliminates *e* to reach $a^2 = ...$ or $a = ...$	M1
	$a = 12$	Cao (not ± 12) unless -12 is rejected	A1
	e = 13/ "12"	Uses their *a* to find *e* or finds *e* by eliminating *a* (Ignore ± here) (Can be implied by a correct answer)	M1
	$x = (\pm)\frac{a}{e}, = \pm\frac{144}{13}$	M1: $(x =)(\pm)\frac{a}{e}$ $\pm$ not needed for this mark nor is *x* and even allow y = $(\pm)\frac{a}{e}$ here – just look for use of $\frac{a}{e}$ with numerical *a* and *e*. A1: $x = \pm\frac{144}{13}$ oe but must be an equation (Do not allow $x = \pm\frac{12}{13/12}$)	M1, A1
			Total 6
	If they use the eccentricity equation for the ellipse ($b^2=a^2(1-e^2)$) allow the M's		

Question Number	Scheme	Marks
2. (a)	$k\,\text{arsinh}\left(\frac{2x}{3}\right)\ (+c)$ or $k\ln[px+\sqrt{(p^2x^2+\frac{9}{4}p^2)}]\ (+c)$	M1
	$\frac{1}{2}\text{ar}\sinh\left(\frac{2x}{3}\right)\ (+c)$ or $\frac{1}{2}\ln[px+\sqrt{(p^2x^2+\frac{9}{4}p^2)}]\ (+c)$	A1
		(2)
(b)	So: $\frac{1}{2}\ln\left[6+\sqrt{45}\right]-\frac{1}{2}\ln\left[-6+\sqrt{45}\right]=\frac{1}{2}\ln\left[\frac{6+\sqrt{45}}{-6+\sqrt{45}}\right]$	M1
	Uses correct limits <u>and</u> combines logs	
	$=\frac{1}{2}\ln\left[\frac{6+\sqrt{45}}{-6+\sqrt{45}}\right]\left[\frac{6+\sqrt{45}}{6+\sqrt{45}}\right]=\frac{1}{2}\ln\left[\frac{(6+\sqrt{45})^2}{9}\right]$	M1
	Correct method to rationalise denominator (may be implied) **Method must be clear if answer does not follow their fraction**	
	$=\ln[2+\sqrt{5}]$ (or $\frac{1}{2}\ln[9+4\sqrt{5}]$)	A1cso
	Note that the last 3 marks can be scored without the need to rationalise e.g. $2\times\frac{1}{2}\left[\ln[2x+\sqrt{(4x^2+9)}]\right]_0^3=\ln(6+\sqrt{45})-\ln 3=\ln(\frac{6+\sqrt{45}}{3})$ M1: Uses the limits 0 and 3 and doubles M1: Combines Logs A1: $\ln[2+\sqrt{5}]$ oe	
		(3)
		Total 5
Alternative for **(a)**	$x=\frac{3}{2}\sinh u\Rightarrow\int\frac{1}{\sqrt{9\sinh^2 u+9}}.\frac{3}{2}\cosh u\,du=k\,\text{arsinh}\left(\frac{2x}{3}\right)\ (+c)$	M1
	$\frac{1}{2}\text{ar}\sinh\left(\frac{2x}{3}\right)\ (+c)$	A1
Alternative for **(b)**	$\left[\frac{1}{2}\text{arsinh}\left(\frac{2x}{3}\right)\right]_{-3}^{3}=\frac{1}{2}\text{arsinh}(2)-\frac{1}{2}\text{arsinh}(-2)$	
	$\frac{1}{2}\ln(2+\sqrt{5})-\frac{1}{2}\ln(\sqrt{5}-2)=\frac{1}{2}\ln(\frac{2+\sqrt{5}}{\sqrt{5}-2})$	M1
	Uses correct limits <u>and</u> combines logs	
	$=\frac{1}{2}\ln(\frac{2+\sqrt{5}}{\sqrt{5}-2}.\frac{\sqrt{5}+2}{\sqrt{5}+2})=\frac{1}{2}\ln(\frac{2\sqrt{5}+4+5+2\sqrt{5}}{5-4})$	M1
	Correct method to rationalise denominator (may be implied) **Method must be clear if answer does not follow their fraction**	
	$=\frac{1}{2}\ln[9+4\sqrt{5}]$	A1cso

Question Number	Scheme		Marks
3.	$(\frac{dx}{d\theta}) = 2\sinh 2\theta$ **and** $(\frac{dy}{d\theta}) = 4\cosh\theta$ Or equivalent correct derivatives		B1
	$A = (2\pi)\int 4\sinh\theta\sqrt{"2\sinh 2\theta"^2 + "4\cosh\theta"^2}\,d\theta$ or $A = (2\pi)\int 4\sinh\theta\sqrt{(1+(\frac{"4\cosh\theta"}{"2\sinh 2\theta"})^2}\,.2\sinh 2\theta\,d\theta$		M1
	Use of correct formula including replacing dx with $"2\sinh 2\theta"d\theta$ if chain rule used. Allow the omission of the 2π here.		
	$A = 32\pi\int \sinh\theta\cosh^2\theta\,d\theta$ $A = 32\pi\int(\sinh\theta + \sinh^3\theta)\,d\theta$		B1
	<u>Completely correct</u> expression for *A* with the square root removed **This mark may be recovered later if the 2π is introduced later**		
	$A = \frac{32\pi}{3}\left[\cosh^3\theta\right]_0^1$	M1: Valid attempt to integrate a correct expression or a multiple of a correct expression – dependent on the first M1 A1: Correct expression	dM1A1
	$= \frac{32\pi}{3}\left[\cosh^3 1 - 1\right]$	M1: Uses the limits 0 and 1 correctly. Dependent on **both** previous M's A1: Cao and cso (no errors seen)	ddM1A1
			(7)
	Example Alternative Integration for last 4 marks		
	$\int\sinh\theta\cosh^2\theta\,d\theta = \int\sinh\theta(1+\sinh^2\theta)\,d\theta = \int(\sinh\theta + \sinh^3\theta)\,d\theta$ $\int(\sinh\theta + \frac{1}{4}\sinh 3\theta - \frac{3}{4}\sinh\theta)\,d\theta = \frac{1}{4}\int(\sinh\theta + \sinh 3\theta)\,d\theta$ $= \frac{1}{4}\cosh\theta + \frac{1}{12}\cosh 3\theta$ **dM1:** $\int\sinh\theta\cosh^2\theta\,d\theta = p\cosh\theta + q\cosh 3\theta$ **A1:** $32\pi\left[\frac{1}{4}\cosh\theta + \frac{1}{12}\cosh 3\theta\right]$		dM1A1
	$A = 8\pi\left[\cosh\theta + \frac{1}{3}\cosh 3\theta\right]_0^1$ $= 8\pi(\cosh 1 + \frac{1}{3}\cosh 3 - \cosh 0 - \frac{1}{3}\cosh 0)$ .... $\frac{32\pi}{3}\left[\cosh^3 1 - 1\right]$	M1: Uses the limits 0 and 1 correctly. Dependent on **both** previous M's A1: Cao	ddM1A1

Question Number	Scheme		Marks
3.	**Alternative Cartesian Approach**		
	$x = 1 + \frac{y^2}{8}$	Any correct Cartesian equation	B1
	$\frac{dx}{dy} = \frac{y}{4}$ or $\frac{dy}{dx} = \frac{\sqrt{2}}{(x-1)^{\frac{1}{2}}}$	Correct Derivative	B1
	$A = \int 2\pi . y\sqrt{(1+\left(\frac{y}{4}\right)^2)}dy$ or $A = \int 2\pi . \sqrt{8}(x-1)^{\frac{1}{2}}\sqrt{(1+\left(\frac{2}{x-1}\right))}dx$		M1
	Use of a correct formula		
	$A = 2\pi \times \frac{2}{3} \times 8\left(1+\frac{y^2}{16}\right)^{\frac{3}{2}}$ or $A = \frac{4\pi\sqrt{8}}{3}(x+1)^{\frac{3}{2}}$		dM1 A1
	M1: Convincing attempt to integrate a relevant expression – dependent on the first M1 but allow the omission of 2π		
	A1: Completely correct expression for A		
	$A = 2\pi \times \frac{2}{3} \times 8(1+\sinh^2 1)^{\frac{3}{2}} - 2\pi \times \frac{2}{3} \times 8$ or $2\pi \times \frac{2}{3} \times \sqrt{8}(1+\cosh 2)^{\frac{3}{2}} - \frac{32\pi}{3}$		ddM1
	Correct use of limits (0 → 4sinh1 for y or 1 → cosh2 for *x*)		
	Use $1+\sinh^2 1 = \cosh^2 1$ to give $\frac{32\pi}{3}\left[\cosh^3 1 - 1\right]$	Use $\cosh 2 = 2\cosh^2 1 - 1$ to give $\frac{32\pi}{3}\left[\cosh^3 1 - 1\right]$	A1

Question Number	Scheme		Marks
4.	$\frac{dy}{dx} = \frac{40}{\sqrt{(x^2-1)}} - 9$	M1: $\frac{dy}{dx} = \frac{p}{\sqrt{(x^2-1)}} - q$ A1: Cao	M1 A1
	Put $\frac{dy}{dx} = \mathbf{0}$ and obtain $x^2 = \dots$ **(Allow sign errors only)**	e.g. $\left(\frac{1681}{81}\right)$	dM1
	$x = \frac{41}{9}$	M1: Square root A1: $x = \frac{41}{9}$ or exact equivalent (not $\pm\frac{41}{9}$)	M1 A1
	$y = 40\ln\{(\frac{41}{9}) + \sqrt{(\frac{41}{9})^2 - 1}\} - "41"$	Substitutes $x = "\frac{41}{9}"$ into the curve and uses the logarithmic form of arcosh	M1
	So $y = 80\ln 3 - 41$	Cao	A1
			Total 7

Question Number	Scheme		Marks
5. (a) (i)&(ii)	$\begin{pmatrix}1&1&a\\2&b&c\\-1&0&1\end{pmatrix}\begin{pmatrix}0\\1\\1\end{pmatrix}=\begin{pmatrix}1+a\\b+c\\1\end{pmatrix}=\lambda_1\begin{pmatrix}0\\1\\1\end{pmatrix}$, and so $a=-1$, $\lambda_1=1$		M1, A1, A1
	M1: Multiplies out matrix with first eigenvector and puts equal to λ_1 times eigenvector. A1 : Deduces *a* = -1. A1: Deduces λ_1 = 1		
	$\begin{pmatrix}1&1&a\\2&b&c\\-1&0&1\end{pmatrix}\begin{pmatrix}1\\0\\-1\end{pmatrix}=\begin{pmatrix}1-a\\2-c\\-2\end{pmatrix}=\lambda_2\begin{pmatrix}1\\0\\-1\end{pmatrix}$, and so $c=2$, $\lambda_2=2$		M1, A1, A1
	M1: Multiplies out matrix with second eigenvector and puts equal to λ_2 times eigenvector. A1: Deduces *c* = 2. A1: Deduces λ_2 = 2		
	$b+c=\lambda_1$ so $b=-1$	M1: Uses $b+c=\lambda_1$ with their λ_1 to find a value for *b* (They must have an equation in *b* and *c* from the first eigenvector to score this mark)	M1A1
		A1: $b=-1$	
	($a=-1$, $b=-1$, $c=2$, $\lambda_1=1$, $\lambda_2=2$)		(8)
(b)(i)	det**P** $=-d-1$	Allow $1-d-2$ or $1-(2+d)$ A correct (possibly un-simplified) determinant	B1
(ii)	$\mathbf{P}^T=\begin{pmatrix}1&2&-1\\1&1&0\\0&d&1\end{pmatrix}$ **or** minors $\begin{pmatrix}1&d+2&1\\1&1&1\\d&d&-1\end{pmatrix}$ **or** cofactors $\begin{pmatrix}1&-2-d&1\\-1&1&-1\\d&-d&-1\end{pmatrix}$ a **correct** first step		B1
	$\dfrac{1}{-d-1}\begin{pmatrix}1&-1&d\\-2-d&1&-d\\1&-1&-1\end{pmatrix}$	M1: Identifiable full attempt at inverse **including reciprocal of determinant**. Could be indicated by at least 6 correct elements.	M1 A1 A1
		A1: Two rows or two columns correct (ignoring determinant) **BUT M0A1A0 or M0A1A1 is not possible**	
		A1: Fully correct inverse	
			(5)
			Total 13

Question Number	Scheme		Marks
6(a)	$I_n=\int_0^4 \underline{x^{n-1}\times x(16-x^2)^{\frac{1}{2}}}\mathrm{d}x$	M1: Obtains $x(16-x^2)^{\frac{1}{2}}$ prior to integration A1: Correct underlined expression (can be implied by their integration)	M1A1
	$I_n=\left[-\frac{1}{3}x^{n-1}(16-x^2)^{\frac{3}{2}}\right]_0^4+\frac{n-1}{3}\int_0^4 x^{n-2}(16-x^2)^{\frac{3}{2}}\mathrm{d}x$		dM1
	dM1: Parts in the correct direction (Ignore limits)		
	$\therefore I_n=\frac{n-1}{3}\int_0^4 x^{n-2}(16-x^2)(16-x^2)^{\frac{1}{2}}\mathrm{d}x$		
	i.e. $I_n=\frac{16(n-1)}{3}I_{n-2}-\frac{n-1}{3}I_n$	Manipulates to obtain at least one integral in terms of I_n or I_{n-2} on the rhs.	M1
	$I_n(1+\frac{n-1}{3})=\frac{16(n-1)}{3}I_{n-2}$	Collects terms in I_n from both sides	M1
	$(n+2)I_n=16(n-1)I_{n-2}$ *	Printed answer with no errors	A1*cso
			(6)
Way 2	$\int_0^4 x^n(16-x^2)^{\frac{1}{2}}\mathrm{d}x=\int_0^4 x^n\frac{(16-x^2)}{(16-x^2)^{\frac{1}{2}}}\mathrm{d}x=\int_0^4\frac{16x^n}{(16-x^2)^{\frac{1}{2}}}\mathrm{d}x-\int_0^4\frac{x^{n+2}}{(16-x^2)^{\frac{1}{2}}}\mathrm{d}x$		
	$=\int_0^4 16x^{n-1}\times x(16-x^2)^{-\frac{1}{2}}\mathrm{d}x-\int_0^4 x^{n+1}\times x(16-x^2)^{-\frac{1}{2}}\mathrm{d}x$		M1A1
	M1: Obtains $x(16-x^2)^{-\frac{1}{2}}$ prior to integration A1: Correct expressions		
	$=\left[-16x^{n-1}(16-x^2)^{\frac{1}{2}}\right]_0^4+16(n-1)\int_0^4 x^{n-2}(16-x^2)^{\frac{1}{2}}\mathrm{d}x$ $-\left(\left[-x^{n+1}(16-x^2)^{\frac{1}{2}}\right]_0^4+(n+1)\int_0^4 x^n(16-x^2)^{\frac{1}{2}}\mathrm{d}x\right)$		dM1
	dM1: Parts in the correct direction on both (Ignore limits)		
	$I_n=16(n-1)I_{n-2}-(n+1)I_n$	Manipulates to obtain at least one integral in terms of I_n or I_{n-2} on the rhs.	M1
	$I_n(1+n+1)=16(n-1)I_{n-2}$	Collects terms in I_n from both sides	M1
	$(n+2)I_n=16(n-1)I_{n-2}$ *	Printed answer with no errors	A1*
Way 3	$\int_0^4 x^n(16-x^2)^{\frac{1}{2}}\mathrm{d}x=\int_0^4 x\times x^{n-1}\frac{(16-x^2)}{(16-x^2)^{\frac{1}{2}}}\mathrm{d}x$	M1: Obtains $x(16-x^2)^{-\frac{1}{2}}$ prior to integration A1: Correct expression	M1A1
	$=\left[-x^{n-1}(16-x^2)(16-x^2)^{\frac{1}{2}}\right]_0^4+\int_0^4(16(n-1)x^{n-2}-(n+1)x^n)(16-x^2)^{\frac{1}{2}}\mathrm{d}x$		dM1
	dM1: Parts in the correct direction (Ignore limits)		
	$I_n=16(n-1)I_{n-2}-(n+1)I_n$	Manipulates to obtain at least one integral in terms of I_n or I_{n-2} on the rhs.	M1
	$I_n(1+n+1)=16(n-1)I_{n-2}$	Collects terms in I_n from both sides	M1
	$(n+2)I_n=16(n-1)I_{n-2}$ *	Printed answer with no errors	A1*

Question Number	Scheme		Marks
(b)	$I_1 = \int_0^4 x\sqrt{(16-x^2)}\mathrm{d}x = \left[-\frac{1}{3}(16-x^2)^{\frac{3}{2}}\right]_0^4 = \frac{64}{3}$	M1: Correct integration to find I_1 A1: $\frac{64}{3}$ or equivalent (May be implied by a later work – they are not asked explicitly for I_1)	M1 A1
	$\frac{64}{3}$ must come from correct work		
	Using $x = 4\sin\theta$: $I_1 = \int_0^{\frac{\pi}{2}} 4\sin\theta\sqrt{(16-16\sin^2\theta)}\,4\cos\theta\,\mathrm{d}\theta = \int_0^{\frac{\pi}{2}} 64\sin\theta\cos^2\theta\,\mathrm{d}\theta$ $= \left[-\frac{64}{3}\cos^3\theta\right]_0^{\frac{\pi}{2}}$ M1: A <u>complete</u> substitution and attempt to substitute <u>changed</u> limits A1: $\frac{64}{3}$ or equivalent		
	$I_5 = \frac{64}{7}I_3,\ I_3 = \frac{32}{5}I_1$	Applies to apply reduction formula twice. First M1 for I_5 in terms of I_3, second M1 for I_3 in terms of I_1 (Can be implied)	M1, M1
	$I_5 = \frac{131072}{105}$	Any <u>**exact**</u> equivalent (Depends on all previous marks having been scored)	A1
			(5)
			Total 11

Question Number	Scheme		Marks
7(a)	$\left(\frac{dx}{d\theta} = -a\sin\theta\right.$ and $\left.\frac{dy}{d\theta} = b\cos\theta\right)$ so $\frac{dy}{dx} = \frac{b\cos\theta}{-a\sin\theta}$		M1 A1
	M1: Differentiates both x and y and divides correctly A1: Fully correct derivative		
	Alternative: M1: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{2x}{a^2} + \frac{2yy'}{b^2} = 0 \Rightarrow y' = -\frac{b^2x}{a^2y} = -\frac{b^2a\cos\theta}{a^2b\sin\theta}$ Differentiates implicitly **and** substitutes for x and y A1: $= -\frac{b\cos\theta}{a\sin\theta}$		
	Normal has gradient $\frac{a\sin\theta}{b\cos\theta}$ *or* $\frac{a^2y}{b^2x}$	Correct perpendicular gradient rule	M1
	$(y - b\sin\theta) = \frac{a\sin\theta}{b\cos\theta}(x - a\cos\theta)$	Correct straight line method using a 'changed' gradient which is a function of θ	M1
	If $y = mx + c$ is used need to find c for M1		
	$ax\sin\theta - by\cos\theta = (a^2 - b^2)\sin\theta\cos\theta$ *		A1
	Fully correct completion to printed answer		
			(5)
(b)	$x = \frac{(a^2 - b^2)\cos\theta}{a}$	Allow un-simplified	B1
	$y = -\frac{(a^2 - b^2)\sin\theta}{b}$	Allow un-simplified	B1
	$\left(= \frac{1}{2}\frac{(a^2 - b^2)^2\cos\theta\sin\theta}{ab}\right)$	$= \frac{1}{4}\frac{(a^2 - b^2)^2}{ab}\sin 2\theta$	M1A1
	M1: Area of triangle is $\frac{1}{2}$"OA"$\times$"OB" and uses double angle formula correctly **A1: Correct expression for the area (must be positive)**		
			(4)
(c)	Maximum area when $\sin 2\theta = 1$ so $\theta = \frac{\pi}{4}$ or 45	Correct value for θ (may be implied by correct coordinates)	B1
	So the point P is at $\left(\frac{a}{\sqrt{2}}, \frac{b}{\sqrt{2}}\right)$ oe $\left(a\cos\frac{\pi}{4}, b\sin\frac{\pi}{4}\right)$ scores B1M1A0	M1: Substitutes their value of θ where $0 < \theta < \frac{\pi}{2}$ or $0 < \theta < 90$ into their parametric coordinates A1: Correct exact coordinates	M1 A1
	Mark part (c) independently		
			(3)
			Total 12

Question Number	Scheme		Marks
8(a)	$(\mathbf{6i}+\mathbf{2j}+\mathbf{12k}).(3\mathbf{i}-4\mathbf{j}+2\mathbf{k})=34$	Attempt scalar product	M1
	$\left\lvert\dfrac{(\mathbf{6i}+\mathbf{2j}+\mathbf{12k}).(3\mathbf{i}-4\mathbf{j}+2\mathbf{k})-5}{\sqrt{3^2+4^2+2^2}}\right\rvert$	Use of correct formula	M1
	$\sqrt{29}$ (not $-\sqrt{29}$)	Correct distance (Allow $29/\sqrt{29}$)	A1
			(3)
(a) Way 2	$\mathbf{r}=(\mathbf{6i}+\mathbf{2j}+\mathbf{12k})+\lambda(3\mathbf{i}-4\mathbf{j}+2\mathbf{k})$ $\therefore\ (6+3\lambda)3+(2-4\lambda)(-4)+(12+2\lambda)2=5$		M1
	Substitutes the parametric coordinates of the line through (6, 2, 12) perpendicular to the plane into the cartesian equation.		
	$\lambda=-1\Rightarrow\ 3,6,10$ or $3\mathbf{i}+4\mathbf{j}-2\mathbf{k}$	Solves for λ to obtain the required point or vector.	M1
	$\sqrt{29}$	Correct distance	A1
(a) Way 3	**Parallel plane** containing (6, 2, 12) is $\mathbf{r}.(3\mathbf{i}-4\mathbf{j}+2\mathbf{k})=34$ $\Rightarrow\dfrac{\mathbf{r}.(3\mathbf{i}-4\mathbf{j}+2\mathbf{k})}{\sqrt{29}}=\dfrac{34}{\sqrt{29}}$	Origin to this plane is $\dfrac{34}{\sqrt{29}}$	M1
	$\Rightarrow\dfrac{\mathbf{r}.(3\mathbf{i}-4\mathbf{j}+2\mathbf{k})}{\sqrt{29}}=\dfrac{5}{\sqrt{29}}$	Origin to plane is $\dfrac{5}{\sqrt{29}}$	M1
	$\dfrac{34}{\sqrt{29}}-\dfrac{5}{\sqrt{29}}=\sqrt{29}$	Correct distance	A1
(b) **For a cross product, if the method is unclear, 2 out of 3 components should be correct for M1**	$\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\2&1&5\\1&-1&-2\end{vmatrix}=\begin{pmatrix}3\\9\\-3\end{pmatrix}$	M1: Attempts $(2\mathbf{i}+1\mathbf{j}+5\mathbf{k})\times(\mathbf{i}-\mathbf{j}-2\mathbf{k})$ A1: Any multiple of **i + 3j – k**	M1A1
	$(\cos\theta)=\dfrac{(\mathbf{3i}-\mathbf{4j}+\mathbf{2k}).(\mathbf{i}+\mathbf{3j}-\mathbf{k})}{\sqrt{3^2+4^2+2^2}\sqrt{1^2+3^2+1^2}}\ \left(=\dfrac{-11}{\sqrt{29}\sqrt{11}}\right)$		M1
	Attempts scalar product of normal vectors including magnitudes		
	52	Obtains angle using arccos (dependent on previous M1)	dM1 A1
	Do not isw and mark the final answer e.g. 90 – 52 = 38 loses the A1		(5)
(c)	$\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&3&-1\\3&-4&2\end{vmatrix}=\begin{pmatrix}2\\-5\\-13\end{pmatrix}$	M1: Attempt cross product of normal vectors A1: Correct vector	M1A1
	$x=0:(0,\ \dfrac{5}{2},\ \dfrac{15}{2}),\quad y=0:(1,\ 0,\ 1),\quad z=0:(\dfrac{15}{13},\ \dfrac{-5}{13},\ 0)$		M1A1
	M1: Valid attempt at a point on both planes. A1: Correct coordinates **May use way 3 to find a point on the line**		
	$\mathbf{r}\times(\mathbf{-2i}+\mathbf{5j}+\mathbf{13k})=-\mathbf{5i}-\mathbf{15j}+\mathbf{5k}$	M1: $\mathbf{r}\times$ dir = pos.vector $\times$ dir **(This way round)** A1: Correct equation	M1A1
			(6)

Question Number	Scheme	Marks
(c) **Way 2**	“$x + 3y - z = 0$” and $3x - 4y + 2z = 5$ uses their cartesian form of and eliminate x, or y or z and substitutes back to obtain two of the variables in terms of the third	M1
	$(x = 1 - \frac{2}{5}y$ and $z = 1 + \frac{13}{5}y)$ or $(y = \frac{5z-5}{13}$ and $x = \frac{15-2z}{13})$ or $(y = \frac{5-5x}{2}$ and $z = \frac{15-13x}{2})$	A1
	Cartesian Equations: $x = \dfrac{y-\frac{5}{2}}{-\frac{5}{2}} = \dfrac{z-\frac{15}{2}}{-\frac{13}{2}}$ or $\dfrac{x-1}{-\frac{2}{5}} = y = \dfrac{z-1}{\frac{13}{5}}$ or $\dfrac{x-\frac{15}{13}}{-\frac{2}{13}} = \dfrac{y+\frac{5}{13}}{\frac{5}{13}} = z$	
	Points and Directions: Direction can be any multiple $(0,\frac{5}{2},\frac{15}{2}), \mathbf{i} - \frac{5}{2}\mathbf{j} - \frac{13}{2}\mathbf{k}$ or $(1,0,1), -\frac{2}{5}\mathbf{i} + \mathbf{j} + \frac{13}{5}\mathbf{k}$ or $(\frac{15}{13}, -\frac{5}{13}, 0), -\frac{2}{13}\mathbf{i} + \frac{5}{13}\mathbf{j} + \mathbf{k}$	M1 A1
	M1:Uses their Cartesian equations correctly to obtain a point and direction A1: Correct point and direction – it may not be clear which is which – i.e. look for the correct numbers either as points or vectors	
	Equation of line in required form: e.g. $\mathbf{r} \times (\mathbf{-2i} + \mathbf{5j} + \mathbf{13k}) = \mathbf{-5i} - \mathbf{15j} + \mathbf{5k}$ Or Equivalent	M1 A1
		(6)
		Total 14
(c) **Way 3**	$\begin{pmatrix} 2\lambda + \mu \\ \lambda - \mu \\ 5\lambda - 2\mu \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix} = 5 \Rightarrow 12\lambda + 3\mu = 5$ M1: Substitutes parametric form of $\prod_2$ into the vector equation of $\prod_1$ A1: Correct equation	M1A1
	$\mu = \frac{5}{3}, \lambda = 0$ gives $(\frac{5}{3}, -\frac{5}{3}, \frac{10}{3})$ $\mu = 0, \lambda = \frac{5}{12}$ gives $(\frac{5}{6}, \frac{5}{12}, \frac{25}{12})$ Direction $\begin{pmatrix} -2 \\ 5 \\ 13 \end{pmatrix}$ M1: Finds 2 points and direction A1: Correct coordinates and direction	M1A1
	Equation of line in required form: e.g. $\mathbf{r} \times (\mathbf{-2i} + \mathbf{5j} + \mathbf{13k}) = \mathbf{-5i} - \mathbf{15j} + \mathbf{5k}$ Or Equivalent	M1A1
	Do not allow ‘mixed’ methods – mark the best single attempt	
	NB for checking, a general point on the line will be of the form: **$(1 - 2\lambda, 5\lambda, 1 + 13\lambda)$**	